AN ISOTOPIC STUDY OF THE ISOSCALAR GIANT MONOPOLE RESONANCE IN
MOLYBDENUM NEAR $N = 50$ SHELL-CLOSURE

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1. Introduction

Giant resonances are extremely collective, high frequency modes of nuclear vibration which are generally described within a collective model, without much regard for the microscopic structure or arrangement of the constituent nucleons. In the broadest sense, one can characterize giant resonances via their quantum numbers corresponding to their multipolarity $\lambda$, spin $\sigma$, and isospin $\tau$. Of interest in this proposed thesis work are the electric and isoscalar modes, which correspond to nucleons oscillating in phase with one another regardless of their spin or isospin projections in response to a weak external field. Within the microscopic model, the resonances are described as coherent superpositions of 1 particle-1 hole excitations across major oscillator shells. The operators inducing the isoscalar transitions for a magnetic substate $\mu$ are given in the long-wavelength approximation to first nontrivial order below:

$$
\mathcal{O}_\mu^0 = \frac{1}{2} \sum_{i=1}^{A} r_i^2, \quad \lambda = 0
$$

$$
\mathcal{O}_\mu^1 = \frac{1}{2} \sum_{i=1}^{A} r_i^3 Y_1^\mu (\Omega_i), \quad \lambda = 1
$$

$$
\mathcal{O}_\mu^\lambda = \sum_{i=1}^{A} r_i^\lambda Y_\lambda^\mu (\Omega_i), \quad \lambda \geq 2
$$

(1)

These give rise to the excitations given below in Table 1. The interpretation of Eqs. (1) is that the operators cause oscillations that can be associated with the geometry of the appropriate spherical harmonics $Y_\lambda^\mu$. Thus, with $\hbar \omega \approx 41A^{-1/3}$ MeV in the harmonic oscillator model, we see that these resonances lie within an overlapping region on the order of $E_{\text{extr}} > 10$ MeV, which poses an experimental complication in disentangling the spectra. One can approach this difficulty by taking advantage of selection rules and decomposing angular distributions to distinguish the various modes. Such methods of dealing with this complication will be discussed in the experimental and data analysis sections of this proposal.

<table>
<thead>
<tr>
<th>Operator</th>
<th>$\lambda$</th>
<th>$2\hbar\omega$</th>
<th>$3\hbar\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>monopole</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dipole</td>
<td>1</td>
<td>$\hbar\omega$</td>
<td>$3\hbar\omega$</td>
</tr>
<tr>
<td>quadrupole</td>
<td>2</td>
<td>$(0\hbar\omega)$</td>
<td>$2\hbar\omega$</td>
</tr>
<tr>
<td>octupole</td>
<td>3</td>
<td>$\hbar\omega$</td>
<td>$3\hbar\omega$</td>
</tr>
<tr>
<td>hexadecapole</td>
<td>4</td>
<td>$(0\hbar\omega)$</td>
<td>$2\hbar\omega$</td>
</tr>
</tbody>
</table>

Table 1. Some possible excitations of the isoscalar multipole operators of Eq. (1). Parenthetical quantities indicate intrashell excitations as opposed to collective oscillations across the shell gaps, which are only possible in open-shell nuclei. [3]

Specifically, we study the compressional mode resonances, one of which is the $\lambda = 0$ isoscalar giant monopole resonance (ISGMR). Inspection of Eqs. (1) gives that this mode corresponds to radially symmetric vibrations of all nucleons in phase with one another, thereby inducing a fluctuation, or “breathing”, of the nuclear density. Thus, the ISGMR is a compressional mode for which the scaling model yields a closed-form relationship with the centroid energy in a given nucleus, $E_{\text{ISGMR}}$, and the nuclear incompressibility of that nucleus, $K_A$:

$$
E_{\text{ISGMR}} = \hbar \sqrt{\frac{K_A}{m \langle r_0^2 \rangle}}
$$

(2)

From measurements of these collective oscillations in finite nuclei, one can predict properties of bulk nuclear matter. In particular, the compressional mode resonances allow for extraction of the nuclear incompressibility, $K_\infty$, which is important.

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insofar that it, along with the energy per nucleon at saturation density, \( \varepsilon \), complete the description of the equation of state for nuclear matter near saturation density \([2, 3]\):

\[
K_\infty := 9 \rho_0^2 \frac{d^2 \varepsilon}{d \rho^2} \bigg|_{\rho = \rho_0}
\]

The current constrained value for \( K_\infty \) is \( 240 \pm 20 \) MeV \([2]\).

**Figure 1.** Equation of state for symmetric nuclear matter using current constrained values for \( K_\infty \), with the blue region characterizing “stiff” nuclear matter that is more incompressible, and red characterizing “soft” nuclear matter which is more easily compressed from saturation density.


\( K_\infty \) is generally calculated using a self-consistent quasiparticle RPA framework using \( K_A \) measurements of “standard” nuclei, such as \(^{90}\text{Zr}\) and \(^{208}\text{Pb}\) \([4]\). The same interactions and density functionals which are calibrated to the ISGMR strength distributions in these nuclei and are successful in reproducing measured ground-state observables have been shown to overestimate the energy of the strength distribution for open-shell nuclei such as tin and cadmium \([2, 4, 5]\). For several years, this has been regarded as an unanswered question in nuclear structure, as the soft incompressibility of neutron rich matter in open-shell nuclei remains something that is not predictable by existing theory \([3]\).

Prior to this study, the group completed an experiment on \(^{90,92}\text{Zr}\) and \(^{92}\text{Mo}\) to investigate claims for an unrelated structure effect on the strength distribution near the double-shell closure by the Texas A&M group \([6]\). The results of the experiment indicate that the ISGMR strength distribution are nearly identical in the nuclei, in contrast to the A&M observation of a sudden jump in the strength distribution of open-shell nuclei, and furthermore, that there is no development of softness immediately adjacent to \(^{90}\text{Zr}\) in the nuclear chart.

The experiment for this proposed thesis work was completed on the \(^{94,96,97,98,100}\text{Mo}\) isotopes at the Research Center for Nuclear Physics (RCNP) at Osaka University with the aim to probe the ISGMR strength distribution in \(^{94,96,97,98,100}\text{Mo}\). By moving away from the shell closure, the aim of the experiment is to determine at what point the open-shell softness observed in Refs. \([5, 7]\) begins to manifest itself.

### 2. Experimental Details

The coupled AVF and ring cyclotrons at RCNP produced 100 MeV/u \( \alpha \) particles which were injected into the high precision mass spectrometer Grand Raiden. The choice of probe stems from the high level density associated in the giant resonance excitation region. Since the resonances lie with excitation energy on the order of 10 MeV, the level densities are large, on the order of \( 10^4 \) MeV\(^{-1}\). Moreover, since the energies are high, many of the states lie beyond the particle decay threshold and hence have larger widths than electromagnetic decays, on the order of several keV \([8]\). This, combined with the overlapping of the collective resonances as shown in Table \([1]\), demands that the probe be chosen such that they couple to as few modes other than those of interest as possible. Since \( \alpha \) particles have neither spin nor isospin projections they do not, to first order, couple to the magnetic or isovector modes of the target nucleus.
model is characterized by an optical potential Satchler and Khoa that a single-folding density dependent optical model is well-suited for analyzing (analyze the data, one must have a suitable optical model with which to perform DWBA calculations. It has been shown by elastic scattering distributions over the range of 5–98◦ sections are maximal. Elastic scattering data were also taken for data at extremely forward angle where, for example, become difficult to separate on angular distributions alone. Thus in these measurements, it is of paramount importance to have where instrumental background is highest.

This is especially critical for these measurements, as the ISGMR has a maximal response at forward angles near 0◦ an accurate cross section extraction for the inelastic excitation region, where the cross sections are small (events which result from instrumental background and other processes result in a uniform distribution in the vertical spectrum shown in Fig. 3). The track reconstructions are then generated using the ray-tracing method. Operating Grand Raiden in vertical focusing mode allows for an expected momentum resolving power of p/Δp ≈ 37000 (neglecting effects such as beam spread), and moreover, for a complete elimination of instrumental background as shown in Fig. 3. Events which correspond to true α-target scattering events are focused at the center of the focal plane while events which result from instrumental background and other processes result in a uniform distribution in the vertical spectrum, as shown in Fig. 3. This allows for a direct measurement of this instrumental background, so that its subtraction permits an accurate cross section extraction for the inelastic excitation region, where the cross sections are small (∼ 100 mb/sr/MeV).

This is especially critical for these measurements, as the ISGMR has a maximal response at forward angles near 0◦, and this is where instrumental background is highest.

The calibration was completed using the energy spectra of 24Mg, where known excitation lines allowed for the determination of the excitation energy of 94–108Mo at each spectrometer angle within the range of 0◦ – 10.5◦, in increments of ∼ 1.5◦. In the giant resonance region of the excitation spectra, as evidenced in Fig. 4, beyond several degrees, multipoles of like-parity become difficult to separate on angular distributions alone. Thus in these measurements, it is of paramount importance to have data at extremely forward angle where, for example, λ = 0 and λ = 2 angular distributions are distinct, and the λ = 0 cross sections are maximal. Elastic scattering data were also taken for 98Mo for the purposes of fitting optical model parameters to elastic scattering distributions over the range of 5◦ ≤ θlab ≤ 30◦.

3. DATA ANALYSIS

The laboratory frame data are converted to the center-of-mass frame using the appropriate relativistic kinematics. To analyze the data, one must have a suitable optical model with which to perform DWBA calculations. It has been shown by Satchler and Khoa that a single-folding density dependent optical model is well-suited for analyzing (α, α′) reactions [3]. This model is characterized by an optical potential

\[ U(r) = -V_{DDG}(r) - iW_{Vol}(r) + V_{Coul}(r), \]
with the shape of the real part described by a realistic nuclear density dependent Gaussian interaction potential given by \[ V_{DDG}(r) = V_V \int d\Omega \int_0^\infty dr' r'^2 \rho(r') f(\rho) v_G(s), \]

with

\[ f(\rho) = 1 - \zeta \rho^b(r'), \]
\[ v_G(s) = V_R \exp\left(-|\mathbf{r} - \mathbf{r}'|^2/a^2\right), \]

with \( \zeta \approx 1.9 \text{ fm}^2, \beta = 2/3, \) and \( t \approx 1.88 \text{ fm} \) being empirical constants given in Ref. [10]. Here, \( \rho(r') \) is the ground-state nuclear density distribution which is taken to be a two parameter Fermi function.

The imaginary potential is taken to be a phenomenological Woods-Saxon given by

\[ W_{\text{vol}}(r) = \frac{W_V}{1 + \exp \left( \frac{r - R}{a} \right)} \]

In total, there are four free parameters, \( V_V, W_V, R, \) and \( a. \) To constrain these parameters, elastic angular distributions were extracted for \(^{98}\text{Mo}\) during the experiment. The DWBA code PTOLEMY was then used for least-\( \chi^2 \) fitting of the optical model parameters to the elastic angular distributions. The predictive power of the parameters set was then tested by using adopted \( B(E2) \) and deformation parameters with the OMP to calculate transition potentials for input to the coupled-channels problem, solved again by PTOLEMY to first order within the DWBA framework. The resulting angular distributions are then compared with the experimental cross sections, as shown in Fig. 4.

Once the predictive power of the optical model is assessed, the inelastic scattering data for excitation energies in the giant resonance region are then analyzed. Although \( \alpha \) particles are relatively light, they transfer angular momentum to the target nucleus in such a way that various multipoles which are overlapping in energy are excited on top of a physical continuum. To disentangle such modes, the standard Multipole Decomposition Analysis (MDA) procedure will be used

in which the total response is decomposed into contributions from the various multipoles [12]:

\[ \frac{d^2\sigma_{\text{exp}}}{dE d\Omega} = \sum_\lambda a_\lambda(E) \frac{d^2\sigma_{\text{DWBA}}}{dE d\Omega}. \]

In this analysis, the physical continuum in the spectra over which the giant resonances are distributed of course may also include processes not relevant to the study (e.g., proton or neutron knockout reactions). These processes, however, do not exhibit any coherency in the multipolarity of their angular distributions and so are absorbed into the multipoles within the MDA, including those higher-multipoles exceeding the value of angular momentum readily transferred by \( \alpha \) particles at the incident beam energy [10, 11].

Sufficient statistics are generally obtained using 1 MeV bins over the excitation region for the MDA. The fractional sum-rule coefficients \( a_\lambda(E) \) are defined such that they describe the fraction of the energy-weighted sum rule (EWSR) for a given multipolarity that is exhausted at an energy \( E. \) This is described in detail in Ref. [11], as is the straightforward conversion from \( a_\lambda(E) \) to the strength function for a particular multipole \( S_\lambda(E). \) For \( \lambda = 0: \)

\[ S_0(E) = \left[ \frac{2 J^2 A (L/2)!}{m} \right] a_0(E) E. \]

From \( S_\lambda(E), \) one can readily extract the moments and moment ratios of the strength distribution [11],

\[ [m_k]_\lambda = \int_0^\infty E^k S_\lambda(E) dE. \]
From these, the moment ratios are calculable and associated with the energies of the resonances, for example,

\[ E_{\text{ISGMR}} = \frac{m_1}{m_0} \lambda = 0. \]

Hence this allows for extraction of \( K_A \) for a given nucleus. From this point onward, collaboration with theorists is necessary in order to refine the existing models to reproduce, simultaneously, \( S_0(E) \) for the molybdenum isotopes in this study as well as those of closed-shell nuclei, such as \(^{90}\text{Zr} \) and \(^{208}\text{Pb} \).

4. SUMMARY OF PROPOSED WORK AND RESEARCH OUTLOOK

The proposed work will be principally focused on studying the softness of the nuclear incompressibility via investigating the response of the isoscalar giant monopole resonance in a series of molybdenum isotopes \(^{94,96,98,100}\text{Mo} \). The study of the systematic changes in structure and location of the centroid response of the energies will allow for insight into the nature of the shell effect on these highly collective modes to determine how the previously documented open-shell softness manifests itself near closed shells.

Furthermore, an additional experiment will be completed as part of this thesis work to investigate the ISGMR in \(^{40,42,44,48}\text{Ca} \), to examine some interesting phenomena reported by Texas A&M in this region of the nuclear chart.

For context, one can examine a macroscopic leptodermous expansion for \( K_A \)

\[ K_A \approx K_{\text{vol}} + K_{\text{surf}} A^{-1/3} + K_T \eta^2 + K_{\text{Coul}} \frac{Z^2}{A^{4/3}} + \ldots \]  

This expression is entirely akin to the semi-empirical expansion of the binding energy per nucleon, but within the context of the nuclear fluid being incompressible. The \( K_{\text{vol}} \) is the saturating volume incompressibility, \( K_{\text{surf}} \) is a correction due to the anisotropy of the nearest neighbor arrangements near the nuclear surface, \( K_{\text{Coul}} \) is a correction due to proton repulsion, and \( K_T \) is a correction for the isospin asymmetry of the nucleus. The understanding is that each of the corrective terms serve to make the nucleus less incompressible than if it were characterized by \( K_{\text{vol}} \sim K_{\infty} \) alone. Various nonrelativistic calculations are in agreement that in Eq. \([14]\), \( K_{\text{surf}} \approx -K_{\infty} \) to a good approximation, with relativistic models giving a larger surface contribution \( K_{\text{surf}} \approx -1.16K_{\infty} \). Thus, one sees that reliable extraction of \( K_A \) and with constrained values of \( K_{\infty} \) and \( K_{\text{Coul}} \), one can extract \( K_T \) via a quadratic fit in \( \eta \). The value of the Coulomb term, \( K_{\text{Coul}} \approx -5.2 \pm 0.7 \text{ MeV} \), is a model independent quantity \([15,16,17]\).

It was established by Li et al., and later by Patel et al. that \( K_T = -550 \pm 100 \text{ MeV} \). It was reported by Texas A&M that the ISGMR energies of \(^{40,44,48}\text{Ca} \) are such that the procedure outlined in Eq. \([13]\) and thereafter results in \( K_T \approx +500 \text{ MeV} \). This is highly controversial and would have massive implications for example, in the equation of state for neutron matter which is critical for input to calculations for highly isospin-asymmetric systems such as neutron stars. The experiment at RCNP will serve to investigate these claims and provide an independent study on the response of the ISGMR in calcium isotopes.

REFERENCES


